STEM Math Boot Camp Challenge

Here are a dozen challenging math problems that any STEM student should be able to solve quickly and easily, IF s/he knows how to use a modern 21st Century STEM Math tool such as Mathematica or Wolfram Alpha.

These problems would be very time consuming, difficult, and sometimes impossible, with the classical "horse and buggy" manual tools taught in classical calculus and differential equation 20th Century courses, i.e. a "Dirty Dozen".

A STEM Math Boot Camp Graduate will be able to solve each of these problems very easily in less than five minutes utilizing a modern tool, Wolfram Alpha.

So, if you can complete this challenge in less than one hour you should be ready to compete with the best educated peers you will have in a good STEM University.

If not, then you will benefit greatly from joining the STEM Math Boot Camp.

You should be able to complete the self paced course of study in the STEM Math Boot Camp in about 15 hours.

The facts are you will learn much more STEM Math in the Boot Camp and these problems will really be an "easy dozen".

Calculus, Differential Equations, Linear Algebra, Complex Numbers and some other math topics are the real impediments to understanding and dealing with Science and Engineering and Technology Models for many ill-prepared students.

Historically, and unfortunately still today, these are hurdles that keep many students from majoring in a STEM subject and becoming a STEM professional, which can be a very lucrative and rewarding career.

Modern tools change all of this, and that is what every aspiring potential STEM student needs to master and that is the Mission of the STEM Math Boot Camp.

Here's your challenge . . .

1. Let $P(x) = x^5 + 4.1x^4 +1.1x^3 – 8.2x^2 - 27.4x - 37.5$
   Find the Roots of P(x), both real and complex, to three significant digits. [This is a Pre-calculus problem that is very difficult without a modern tool.]

2. Find the relative Maximum and Minimum Points of P(x), and find the intervals where P(x) is increasing and decreasing.
   [This is a Differential Calculus Problem.]

3. Find the points of inflection of P(x) and the Concavity intervals and graph P(x). [This is a Differential Calculus Problem.]

4. Let $F(x) = x^2 + .5\text{Sin}(10x)$. Graph F(x) from x = 1 to 3, Find the Arc Length of this graph, and find the area beneath the graph from 1 to 3. [This is an Integral Calculus Problem.]

5. Rotate F(x) from problem #4 about the x-axis and find both the Volume and Surface Area of the Solid of Revolution. [This is an Integral Calculus Problem.]

6. Let $G(x) = \text{Sin}(x^2)$. Find the anti-derivative of G(x) and the Area under the graph of G(x) from x = -1.5 to 1.5 [This is an Integral Calculus Problem that would probably not be given in a classical calculus course since there is not an anti-derivative of G(x) consisting of the standard functions, and thus one could not easily apply the Fundamental Theorem of Calculus.]

7. Find the point of intersection of the three planes:
   $3.1x + 4.3y - 7.6z = 5.2$, $2.7x – 3.4y + 5.1z = -6.9$, $-0.9x + 4.2y - 3.8z = 8.7$
   [This is a Linear Algebra problem you should be able to solve in about two minutes with a modern tool.]

8. Let $F(x) = 1.9/(3.1 + 2.7x^2)$, Find a polynomial, $P(x)$, of degree 8 which is the best approximation of $F(x)$ at $x = 0$. Plot both $F(x)$ and $P(x)$ and observe that $P(x)$ is a very good approximation of $F(x)$ from $-.5<x<.5$ [This is an advanced Differential Calculus Problem that would be very time consuming classically, but very easy with a modern tool.]

9. y is a function of x. Find the solution to the differential equation: $y'' + y' + y = \cos(x)$, with initial conditions $y(0) = .5$, and $y'(0) = .3$ [This is a Differential Equation of order 2.]

10. y is a function of t. Find the solution to the differential equation: $y' + y + e^t = 0$, $y(1) = 2$ [This is a Differential Equation of order 1.]

11. $x^2 + y^3 - (xy)^2 = 0$ implicitly defines three functions y of x. What is the asymptotic behavior of these three functions as $x \to \infty$ and $x \to -\infty$? Also, graph these three functions and their asymptotes.

12. What is the length of the curve defined parametrically (sin(t),cos(3t)) from t = 0 to Pi? Graph the curve.

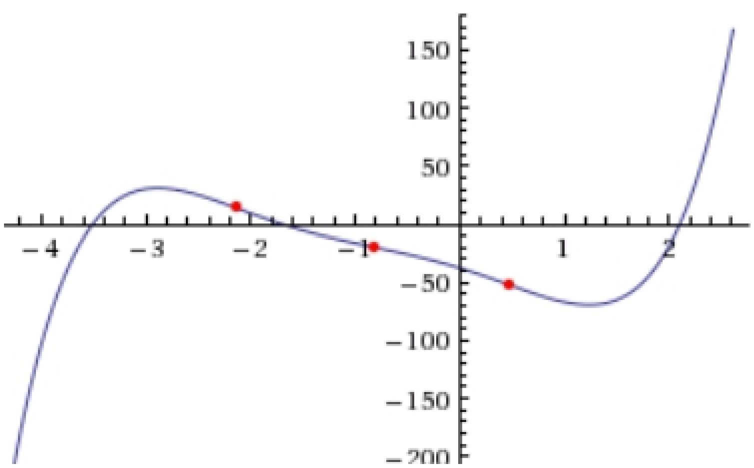
Answers:

1. **-3.51, -1.700, +2.10, -0.498 + 1.66i, -0.498 - 1.66i**

2. **(-2.89, 31.06) is a Maximum, (1.24, -69.4) is a Minimum. P(x) is increasing in the intervals (-∞, -2.89) and (1.24, +∞), and decreasing in the interval (-2.89, 1.24)**

3. **(-2.12, 13.3) and (-0.813, -19.8) and (0.475, -52.02) are the three inflection points and P(x) is concave down in the intervals (-∞, -2.12) and (-0.813, 0.475) and concave up in the intervals (-2.12, -0.813) and (0.475, ∞)**

   The Graph is below and the points of inflection are given on it.



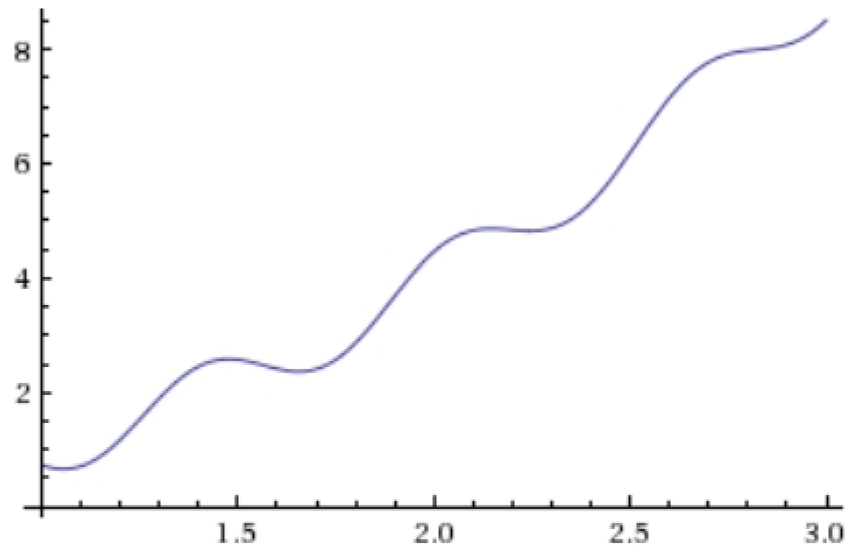
4. **The Arc Length of the Graph of F(x) from 1 to 3 is:**

$$\int_1^3 \sqrt{1 + (2\,x + 5.\cos(10\,x))^2}\;dx \approx 8.9013!$$

The Area under the Graph of F(x) from x = 1 to 3 is:

$$\int_1^3 \left(x^2 + 0.5\sin(10\,x)\right) dx = 8.617$$

Here is the Graph of F(x)



5. **Volume of Solid of Revolution of F(x) about the x axis:**

$$\int_1^3 \pi \left(x^2 + 0.5\sin(10\,x)\right)^2 dx = 152.017$$

Surface Area of Solid of Revolution of F(x) about the x axis:

$$\int_1^3 2\,\pi \left|x^2 + 0.5\sin(10\,x)\right| \sqrt{1 + (2\,x + 5.\cos(10\,x))^2}\;dx = 247.897$$

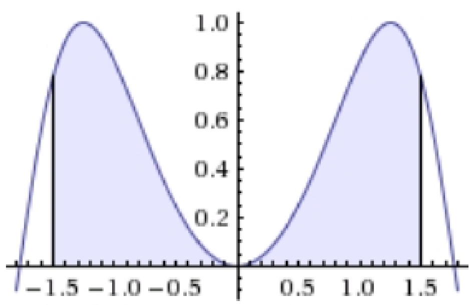
6. **The anti-derivative of $Sin(x^2)$ is a Special Function called a Fresnal Integral:**

$$\int \sin(x^2)\,dx = \sqrt{\frac{\pi}{2}}\, S\left(\sqrt{\frac{2}{\pi}}\, x\right) + \text{constant}$$

The Area under the graph of $Sin(x^2)$ from -1.5 to 1.5 is:

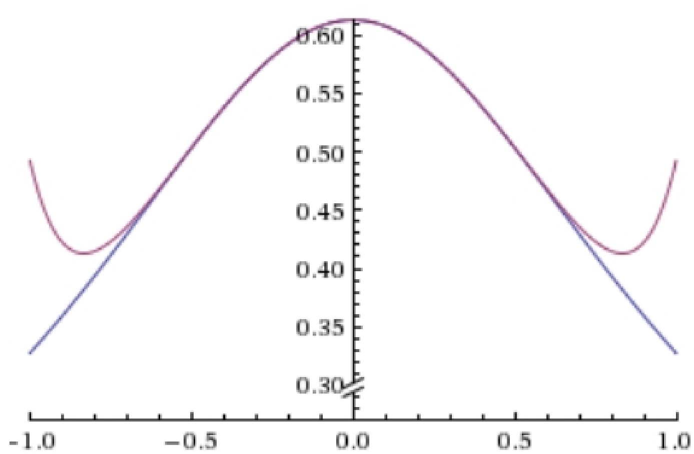
$$\int_{-1.5}^{1.5} \sin(x^2)\,dx = 1.55648$$

Visual representation of the integral:



7. **x = -0.451, y = 2.44, z =.510**

8. **$P(x) = 0.353x^8 – 0.405x^6 + 0.465x^4 – 0.534x^2 +0.613$**



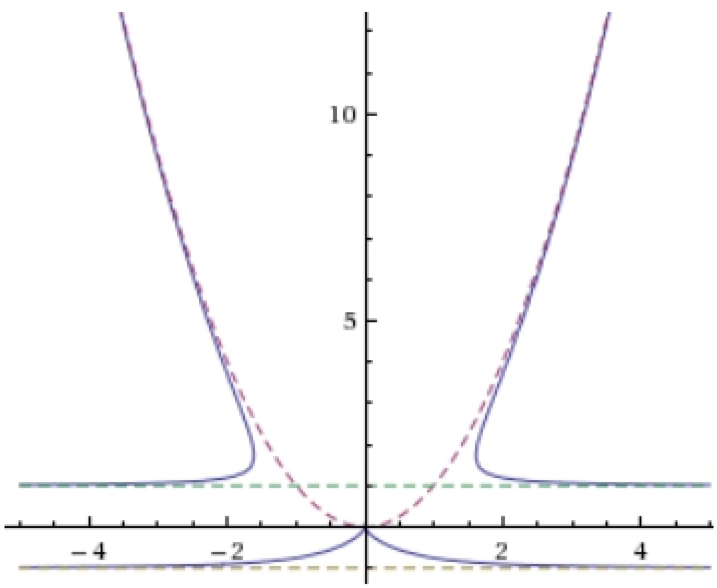
9. **The solution of the differential equation is:**

$$y(x) = \sin(x) - 0.519615\, e^{-x/2} \sin\left(\frac{\sqrt{3}\, x}{2}\right) + 0.5\, e^{-x/2} \cos\left(\frac{\sqrt{3}\, x}{2}\right)$$

10. **The solution of the differential equation is:**

$$y(t) = \frac{1}{2}\, e^{-t} \left(-e^{2t} + 4\, e + e^{2}\right)$$

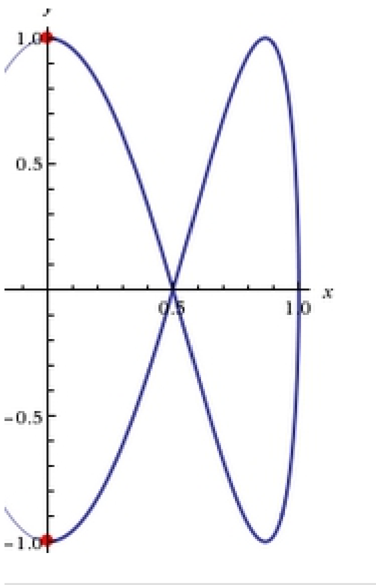
11. **One function approaches y = x^2 asymtotically in both directions, and a second function approaches y = +1 in both directions, and the third function approaches y = -1 in both directions. Notice the symmetry.**



12. **The length is:**

$$\int_0^{\pi} \sqrt{\cos^2(t) + 9\sin^2(3t)}\, dt \approx 6.5327$$

The Graph is:



The tools you will learn to use as a STEM professional will solve much more difficult math problems very easily. These modern tools will, in fact, enable the STEM professional to solve problems that would be simply intractable without the modern tools.

It is IMPERATIVE that a 21st Century STEM student learns and masters the modern tools of STEM.

That is the Mission of Triad Math's STEM Math Boot Camp.

For more information, please visit www.STEMMathMadeEasy.com