RELATED RATES (147–155)

PROBLEM 24

TWO FUNCTIONS OF TIME

$A(t)$ $B(t)$

1) FIND AN EQUATION RELATING THEM

GEOMETRY, ALGEBRA, TRIG.

2) (a) FIND $A'(t)$ IN TERMS OF

$A(t)$, $B(t)$, $B'(t)$

AND

(b) FIND $B'(t)$ IN TERMS OF

$A(t)$, $B(t)$, $A'(t)$

CALCULUS (DERIVATIVE) + ALGEBRA

3) GIVEN $A(t_1)$, $B(t_1)$, $B'(t_1)$

FIND $A'(t_1)$

t_1 IS UNKNOWN - YET COMMON

PAY ATTENTION TO UNITS WHEN POSSIBLE [GOOD CHECK]

~~WHEN~~ OR

GIVEN $A(t_1)$, $B(t_1)$, $A'(t_1)$

FIND $B'(t_1)$

THIS JUST SOLVING A LINEAR ALGEBRA EQUATION

- EASY -

WE CAN USE ANY TWO SYMBOLS FOR THE TWO FUNCTIONS

$A(t)$ AREA $V(t)$ VOLUME

$h(t)$ HEIGHT $\theta(t)$ ANGLE

1) $y(t) = x^2(t) + 3$ $x^2(t) = [x(t)]^2$

2) $A(t) = \pi r^2(t)$

3) $V(t) = \frac{4}{3}\pi r^3(t)$

4) $x^2(t) + 6^2 = s^2(t)$

5) $\tan(\theta(t)) = \frac{s(t)}{2000}$

6) $7^2 = 3^2 + x^2(t) - 2 \cdot 3 \cdot x(t) \cdot \cos(\theta(t))$

#23) $V(t) = 18\, h^2(t)$

#24) $V(t) = 6\, h^2(t)$

1) $y'(t) = 2\,x(t) \cdot x'(t)$

$$x'(t) = y'(t) / 2\,x(t)$$

2) $A'(t) = \pi \cdot 2 \cdot r(t) \cdot r'(t)$

$$r'(t) = A'(t) / 2\pi\, r(t)$$

3) $V'(t) = \frac{4}{3}\pi \cdot 3\, r^2(t) \cdot r'(t)$

$$r'(t) = \frac{V'(t)}{4\pi\, r^2(t)}$$

4) $2\,x(t) \cdot x'(t) = 2\, s(t) \cdot s'(t)$

$$x'(t) = \frac{s(t)\, s'(t)}{x(t)}$$

$$s'(t) = \frac{x(t)\, x'(t)}{s(t)}$$

UNITS — KEEP TRACK

3) $r(t)$ ft $\quad V(t)$ ft^3

$V'(t)$ $\mathrm{ft}^3/\mathrm{min}$ $\quad r'(t)$ ft/min

$$V'(t) = \frac{4}{3}\pi \cdot 3\, r^2(t)\, r'(t)$$

$$\frac{\mathrm{ft}^3}{\mathrm{min}} \qquad \mathrm{ft}^2 \quad \frac{\mathrm{ft}}{\mathrm{min}}$$

$$\frac{\mathrm{ft}}{\mathrm{min}} \quad r'(t) = \frac{V'(t)}{4\pi r^2(t)} \quad \frac{\mathrm{ft}^3/\mathrm{min}}{\mathrm{ft}^2} = \frac{\mathrm{ft}}{\mathrm{min}}$$

THIS IS GOOD WAY TO SPOT A MISTAKE!

GIVEN KNOWN'S FIND UNKNOWN

#3 GIVEN $V'(t_1) = 4.5\ ft^3/MIN$

$r(t_1) = 2\ ft$

FIND $r'(t_1)$

$$r'(t_1) = \frac{V'(t_1)}{4\pi r^2(t_1)} = \frac{4.5}{4\pi \cdot 2^2} \frac{ft^3/MIN}{ft^2}$$

$$= \frac{9}{32\pi} \cdot ft/MIN$$

#23 (b) $V'(t_1) = \frac{1}{4}\ m^3/MIN$ $\quad h(t_1) = 1\ m$

$$V'(t) = 18 \cdot 2 \cdot h(t) \cdot h'(t)$$

$$h'(t_1) = \frac{V'(t_1)}{\underset{m}{36}\ \underset{m}{h(t_1)}} = \frac{\frac{1}{4}}{36 \cdot 1} \frac{m^3/MIN}{m^2}$$

NOTE:

$$V(t) = \underbrace{18}_{m} \underset{m^2}{h^2(t)}\ m^3$$

$$= \frac{1}{144}\ m/MIN$$

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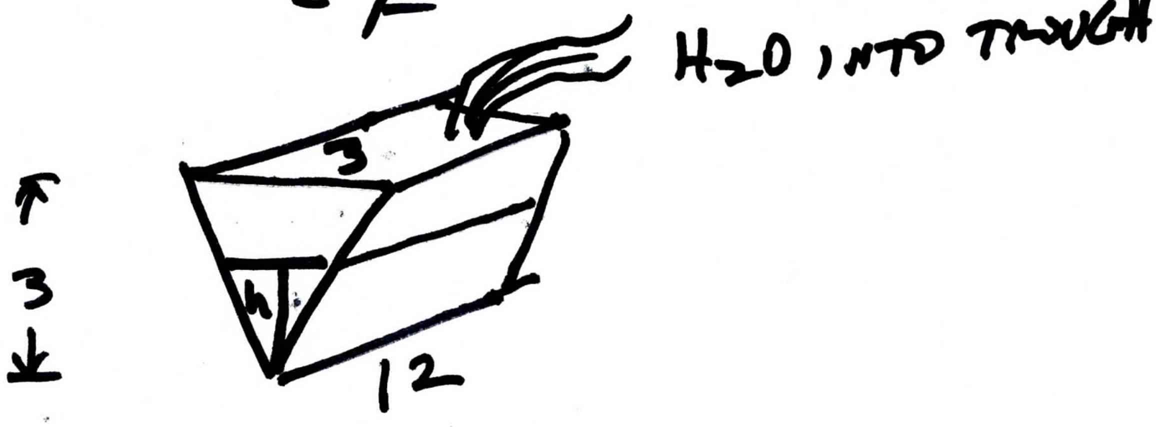

VOLUME OF WATER IN TROUGH HEIGHT $h(t)$

$$V(t) = 12 \times A(t) = 12 \cdot \tfrac{1}{2}\, b(t) \cdot h(t) = \tfrac{1}{2}\, h(t)^2 \times 12$$

ft ft^2

GEOMETRY

$$V(t) = 6\, h(t)^2$$

ft^3 ft ft^2

DIFF + ALG.

$$V'(t) = 12\, h(t) \cdot h'(t)$$

ft^3/min ft ft ft/min

$$h'(t) = \frac{V'(t)}{12 \cdot h(t)}$$

#24(a) $V'(t) = 2\ ft^3/min$ $h(t) = 1\ ft$

$$h'(t) = \frac{V'(t)}{12\ h(t)} = \frac{2}{12 \cdot 1} \frac{ft^3/min}{ft \cdot ft} = \frac{1}{6}\ ft/min$$

24(b) $h'(t) = \frac{3}{8}\ in/min$ $h(t) = 2$ (in or ft?)

ASSUME ft

THEN MUST EXPRESS $h'(t)$ IN ft/min

$1\ in = \frac{1}{12}\ ft$

$h'(t) = \frac{3}{8} \times \frac{1}{12}\ ft/min$

$h(t) = 2$

$$V'(t) = 12 \cdot \underset{ft}{h(t)} \cdot \underset{ft/min}{h'(t)}$$

ft ft ft/min

$$= 12 \cdot 2 \cdot \frac{3}{8} \cdot \frac{1}{12}\ ft^3/min$$

$$= \frac{3}{4}\ ft^3/min$$